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Geometric phase in a composite system subjected to decoherence

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Abstract. In this paper, we investigate the geometric phase of a composite system which is composed of two spin-$\frac{1}{2}$ particles driven by a time-varying magnetic field. Firstly, we consider the special case that only one subsystem driven by time-varying magnetic field. Using the quantum jump approach, we calculate the geometric phase associated with the adiabatic evolution of the system subjected to decoherence. The results show that the lowest order corrections to the phase in the no-jump trajectory is only quadratic in decoherence coefficient. Then, both subsystem driven by time-varying magnetic field is considered, we show that the geometric phase is related to the exchange-interaction coefficient and polar angle of the magnetic field.

PACS. 03.65.Vf Phases: geometric; dynamic or topological – 03.65.Yz Decoherence; open systems; quantum statistical methods

QICS. 03.90.+m (Other) mathematical aspects of composite quantum systems – 02.40.+d Interaction with environment and decoherence

1 Introduction

The concept of geometric phase was firstly discussed by Pancharatnam [1] in his study of interference of classical light in distinct states of polarization. Geometric phases in quantum theory have attracted great interest since the seminal work of Berry [2], in which he demonstrated that a quantum-mechanical system which underwent an adiabatic cyclic evolution acquired a geometrical phase factor in addition to the dynamical one. Simon [3] gave a simple geometric interpretation of Berry phase in the language of differential geometry and fibre bundles. After that, this important notion has become an interesting subject and was a subject of interest in many different aspects, which has led to many different generalizations and applications [4–9]. In reference [10] the authors showed that, in a nonlinear Jaynes-Cummings model [11–14], how to transmute the statistics of the system via the adiabatic geometric phase. An important reason for the interest in the concept of geometric phase is its relevance to geometric quantum computation [15]. Indeed, it is believed that the purely geometric characteristic of such phases potentially provides robustness against certain sources of noise [16].

Geometric phases are useful in the present of quantum computing as a tool to achieve fault tolerance [16,17]. However, practical implementations of computing are always done in the presence of decoherence. Thus, the geometric phase of the open system has attracted many people's interest, The first general approach is Uhlmanns mathematical method which is based on a purification of mixed states [18]. Gamliel and Freed [19] discussed geometrical phase in the context of dissipative evolution of an interacting spin system, governed by the stochastic Liouville equation. Tong et al. [20] proposed a kinematic approach to the mixed state geometric phase in nonunitary evolution and demonstrated that the proposed geometric phase for nonunitarily evolving mixed states is experimentally testable in interferometry. Nazri et al. [21] considered the effects of certain forms of decoherence applied to geometric phase quantum gates and quantified the loss of entanglement as a function of decoherence. Chiara et al. [22] studied the behavior of geometric phase under some typical error sources such as stochastic classical fluctuations to the driving fields and demonstrated that the geometrical aspects of Berry phase can reduce random classical fluctuations, and Carollo and his coworker [23] investigated the geometric phase of a spin-$\frac{1}{2}$ particle interacting with a driving quantized magnetic field which is subjected to decoherence. In reference [24], it has been shown that coupling to environment induces some geometric and non-geometric corrections to the geometric phase of a spin-half system that is under an adiabatically slow rotating magnetic field. we focus on the quantum trajectory approach to the open system geometric phase. Bassi and Ippoliti [25] analyzed geometric phase of open system

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and noted that the definition of geometric phase which related to the type of stochastic unraveling of the master equation may have difficulty. Sjöqvist [26] focused on the quantum trajectory approach to the open system geometric phase and pointed out that the average over the phase factors failed to reflect the geometry of the open system evolution itself.

We are not aware of any studies in which two coupling spin-$\frac{1}{2}$ particles driven by an external classical magnetic field subjected to decoherence, so this is precisely the aim of this paper. In this present paper, we investigate the geometric phase of two coupled spin-$\frac{1}{2}$ particles driven by classical magnetic field, the results show that the geometric phase of the composite system is related to the exchange-interaction coefficient and the polar angle of the magnetic field, then we calculate and analyze the geometric phase associated with the evolution of the composite subjected to decoherence through a quantum-jump approach.

The remainder of this present paper is organized as follows. In Section 2, we briefly review the scheme which is used to study the geometric phase of the open system. In Section 3. the theoretical model is described and the eigenvalue, eigenvector and geometric phase of the system are presented. Then, the influence of exchange-interaction coefficient and the polar angle of the magnetic field on geometric phase are analyzed and discussed. Next, some numerical analysis are given. In Section 4, the lowest corrections of the geometric phase for our system are given when the decoherence effect are considered. Finally, we conclude with a brief discussion and summary in Section 5.

2 Calculating of method

Let us consider a system described by the density operator ρ and a Hamiltonian H. To investigate the effects of decoherence on the geometric evolution of states described by this system, we employ the quantum jump approach, in the Markovian approximation, the master equation reads ($\hbar = 1$)

$$\dot{\rho} = \pounds \frac{1}{i}\left[\mathrm{H}, \rho\right] - \frac{1}{2}\sum_{k=1}^{n}\left\{\Gamma_k^{\dagger}\Gamma_k\rho + \rho\Gamma_k^{\dagger}\Gamma_k - 2\Gamma_k\rho\Gamma_k^{\dagger}\right\}. \quad (1)$$

The commutator generates, through the Hamiltonian H, the coherent part of the evolution and the second part represents the effect of the reservoir on dynamics of the system. The action of each Γ_k amounts to a different decohering process. Assume that the system has no decay, the geometric phase for the "no-jump" trajectory for the master equation (1) in the continuous limit is given by [5]

$$\gamma^0 = \int_0^T \frac{\left\langle \psi^0\left(t\right)\right| H \left|\psi^0\left(t\right)\right\rangle}{\left\langle \psi^0\left(t\right) \mid \psi^0\left(t\right)\right\rangle} dt - \arg\left\{\left\langle \psi^0\left(T\right) \mid \psi^0\left(0\right)\right\rangle\right\}, \quad (2)$$

where

$$i\frac{d}{dt}\left|\psi^0\left(t\right)\right\rangle = \tilde{H}\left|\psi^0\left(t\right)\right\rangle, \quad \left|\psi^0\left(0\right)\right\rangle = \left|\psi_0\right\rangle, \quad (3)$$

and $\tilde{H}$ is a non-Hermitian effective Hamiltonian which is given by

$$\tilde{H} = H - \frac{i}{2}\sum_{k=1}^{n}\Gamma_k^{\dagger}\Gamma_k. \quad (4)$$

3 Adiabatic geometric phase

Let us consider two spin-$\frac{1}{2}$ particles are coupled by a uniaxial exchange interaction in the z direction, which are driven by a time-varying magnetic field $\boldsymbol{B}(t)$, with the Hamiltonian ($\hbar = 1$) [27]

$$\hat{H}(t) = 4J\hat{\sigma}_1^z \otimes \hat{\sigma}_2^z + \mu\boldsymbol{B}(t)\cdot(\hat{\sigma}_1 + \hat{\sigma}_2), \quad (5)$$

where σ_1^z, σ_2^z are Pauli matrices, and the indices 1 and 2 indicate the corresponding subsystems. The presence of a time-dependent external magnetic field is considered to be $B_0\boldsymbol{n}(t)$ with the unit vector $\boldsymbol{n}(t) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. The first part of the Hamiltonian describes the exchange interaction (spin-spin coupling) with exchange-interaction coefficient J (assumed positive without loss of generality). μ is the gyromagnetic ratio assumed to be equal for the two spins and $\sigma_\varepsilon = (\sigma_\varepsilon^x, \sigma_\varepsilon^y, \sigma_\varepsilon^z)$ is the εth spin operator ($\varepsilon = 1, 2$) composed of the Pauli matrices.

It is note that the eigenstates of the system are entangled, so it is difficult to analytically solve this Schrödinger equation. Thus, we firstly consider the simple case that only one subsystem driven by the magnetic filed, and analytically solve Schrödinger equation and obtain the geometric phase. Then, we introduce numerical calculus method to the case that both subsystem driven by the magnetic filed, and analyze and discuss the properties of geometric phase of the composite system.

If we assume that only one subsystem driven by the magnetic field, the Hamiltonian can be reduced to

$$\hat{H}(t) = 4J\hat{\sigma}_1^z \otimes \hat{\sigma}_2^z + \mu\boldsymbol{B}(t)\cdot\hat{\sigma}_1. \quad (6)$$

The eigenvalues E_i of this Hamiltonian can be found simply as follows

$$E_{1,2} = \mp\sqrt{\eta^2 - 2\eta\cos(\theta) + 1}, \quad (7)$$

$$E_{3,4} = \mp\sqrt{\eta^2 + 2\eta\cos(\theta) + 1}, \quad (8)$$

and the corresponding instantaneous eigenvectors $(\mid \Psi_i\left(t\right)\rangle)$ of this Hamiltonian

$$|\Psi_{1,2}\rangle = \frac{1}{\sqrt{M_{1,2}}}[(\cos(\theta) - \eta + E_{1,2})e^{-i\phi}|e, g\rangle + \sin(\theta)|g, g\rangle], \quad (9)$$

$$|\Psi_{3,4}\rangle = \frac{1}{\sqrt{M_{3,4}}}[(\cos(\theta) + \eta + E_{3,4})e^{-i\phi}|e, e\rangle + \sin(\theta)|g, e\rangle], \quad (10)$$

with

$$M_{1,2} = |[\cos(\theta) - \eta + E_{1,2}]e^{-i\phi}|^2 + \sin^2(\theta), \quad (11)$$

$$M_{3,4} = |[\cos(\theta) + \eta + E_{3,4}]e^{-i\phi}|^2 + \sin^2(\theta). \quad (12)$$

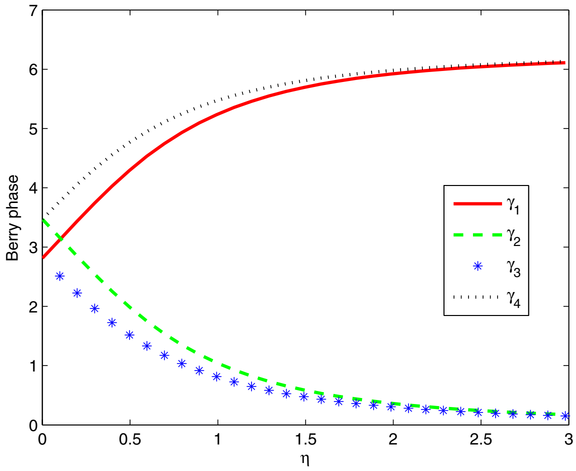

Fig. 1. (Color online) Geometric phase as a function of η with a fixed θ ($\theta = 14\pi/30$).

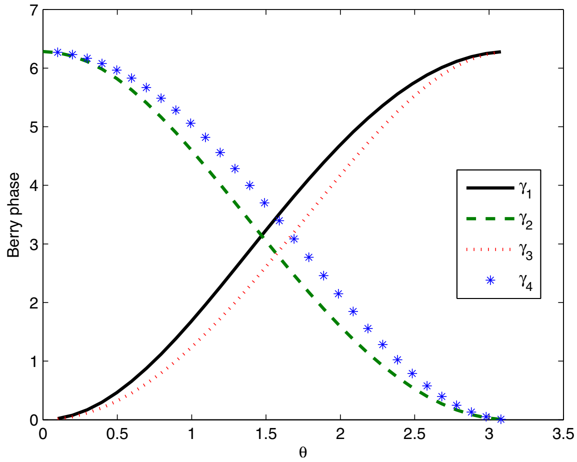

Fig. 2. (Color online) Geometric phase as a function of polar angle of magnetic filed θ with a fixed η ($\eta = 0.1$).

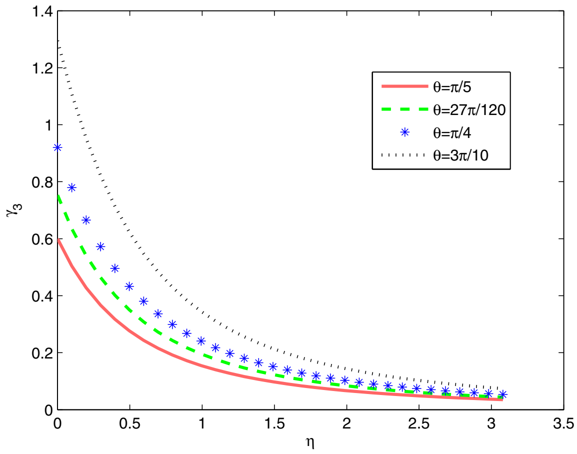

Fig. 3. (Color online) Geometric phase γ_3 as a function of η with different polar angle θ of magnetic field.

where η is related to exchange-interaction coefficient J ($\eta = \frac{4J}{\mu B_0}$). Assuming the system undergoes adiabatic and cyclic evolution starting with an initial state $|\Psi_j(t)\rangle$ ($j = 1,2,3,4$), keeping θ as a constant and changing ϕ slowly from 0 to 2π ($\phi(T) = 2\pi$), the Berry phase generated is calculated as follows:

$$\gamma_j = i\int_0^T dt\langle\Psi_j(t)|\frac{d}{dt}|\Psi_j(t)\rangle. \tag{13}$$

Substituting equations (9) and (10) into equation (13), the explicit expressions of equation (13) are

$$\gamma_{1,2} = 2\pi\frac{[\cos(\theta) - \eta + E_{1,2}]^2}{[\cos(\theta) - \eta + E_{1,2}]^2 + \sin^2(\theta)}, \tag{14}$$

$$\gamma_{3,4} = 2\pi\frac{[\cos(\theta) + \eta + E_{3,4}]^2}{[\cos(\theta) + \eta + E_{3,4}]^2 + \sin^2(\theta)}. \tag{15}$$

In the special case where the exchange interaction constant $J = 0$, the eigenvalues of the system are reduced into $E_\pm = \pm 1$ (in units of $\frac{\mu B_0}{4}$), with the corresponding instantaneous eigenstates $\Psi_+(t) = [\cos(\frac{\theta}{2})e^{-i\phi}|e\rangle + \sin(\frac{\theta}{2})|g\rangle] \otimes |g\rangle$ and $\Psi_-(t) = [-\sin(\frac{\theta}{2})e^{-i\phi}|e\rangle + \cos(\frac{\theta}{2})|g\rangle] \otimes |g\rangle$ and the geometric phases of the system reduce to the well-known results $\gamma_+ = (1+\cos(\theta))\pi$ and $\gamma_- = (1-\cos(\theta))\pi$. This results can be easily accepted. There are no interactions between the subsystem 1 and subsystem 2 when $J = 0$. In other words, the subsystem 1 can freely evolve and it should make no effects on any behaviors of subsystem 2 when the composite system is prepared in a separable state. So the results should reduce to one subsystem case when the other subsystem acquires no geometric phase. Some elegant paper [28] have demonstrated similar results for geometric phase.

Now we give a simple analysis for the influence of exchange-interaction coefficient η (in units of $\frac{\mu B_0}{4}$) on the geometric phase. In Figure 1, we plot the geometric phase as a function of exchange-interaction coefficient η with a fixed θ ($\theta = 14\pi/30$). It is easily seen from Figure 1 that the geometric phases tend to zero (or 2π) when the exchange-interaction coefficient increase continuously ($\eta \to \infty$). Figure 2 shows the geometric phase dependence on the polar angle of magnetic field θ, when we have a fixed exchange-interaction coefficient η ($\eta = 0.1$). As can be seen, the geometric phases also tend to zero (or 2π) when θ increase from 0 to π. In order to further explore the influence of exchange-interaction coefficient η on the geometric phase, we give the corresponding curves in Figures 3 and 4. The specific results are as follows. We present geometric phase γ_3 as a function of exchange-interaction coefficient η with different θ in Figure 3. From Figure 3, we find that the geometric phase γ_3 tends to zero with increasing η and geometric phase γ_3 more quickly reduce to zero when θ continuously decrease. We also plot in Figure 4 geometric phase as a function of θ with different fixed η. It is interesting to compare the change tendency of the geometric phase between $\eta = 0$ and $\eta \neq 0$ ($\eta = 0.1$ or $\eta = 0.3$). Figure 4 clearly shows that geometric phase more slowly tends to zero when $\eta = 0$ than when $\eta \neq 0$.

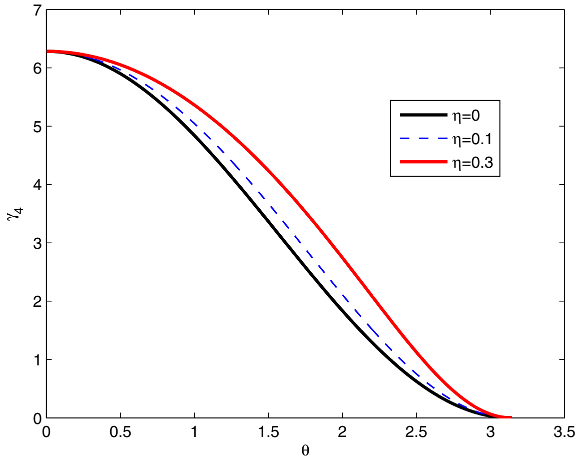

Fig. 4. (Color online) Geometric phase γ_4 as a function of polar angle of magnetic filed θ with different fixed η.

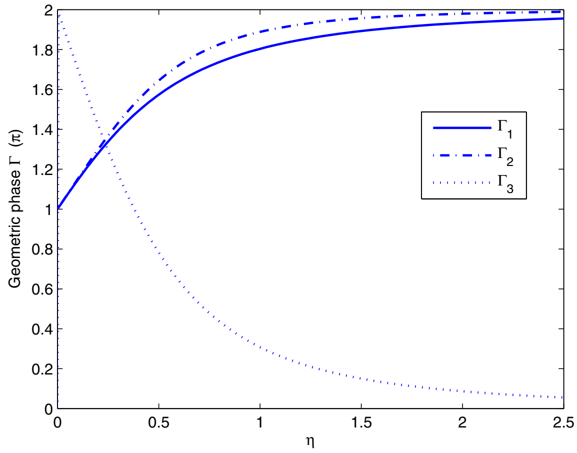

Fig. 5. Geometric phase Γ as a function of η with a fixed angle $\theta = \pi/3$ of magnetic field.

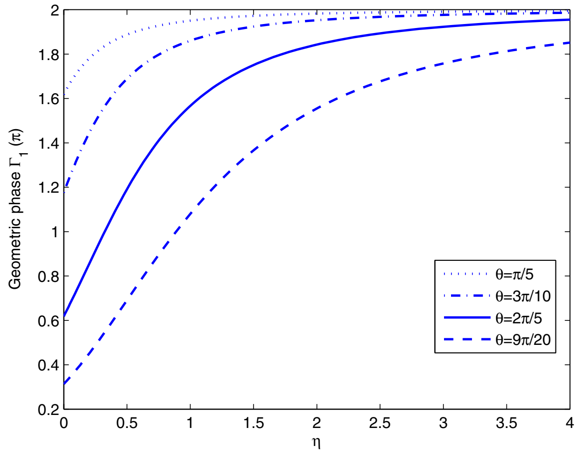

Fig. 6. Geometric phase Γ_1 as a function of η with different polar angle θ of magnetic field.

Then, we calculate the Geometric phase of Hamiltonian (5) by using numerical calculus method. In a space spanned by Φ_1, Φ_2, Φ_3, Φ_4 ($\Phi_1 = |g\rangle|g\rangle$; $\Phi_2 = \frac{\sqrt{2}}{2}(|g\rangle|e\rangle + |e\rangle|g\rangle)$; $\Phi_3 = |e\rangle|e\rangle$; $\Phi_4 = \frac{\sqrt{2}}{2}(|g\rangle|e\rangle - |e\rangle|g\rangle)$) where $|g\rangle$ and $|e\rangle$ stand for spin down and spin up, respectively. The Hamiltonian (5) can be expressed in the block-matrix form [7]

$$H = \begin{pmatrix} H_c(t) & 0 \\ 0 & -\eta \end{pmatrix},$$

with

$$H_c = \begin{pmatrix} \eta - \cos(\theta) & \frac{\sqrt{2}}{2}\sin(\theta)e^{i\phi} & 0 \\ \frac{\sqrt{2}}{2}\sin(\theta)e^{-i\phi} & -\eta & \frac{\sqrt{2}}{2}\sin(\theta)e^{i\phi} \\ 0 & \frac{\sqrt{2}}{2}\sin(\theta)e^{-i\phi} & \eta + \cos(\theta) \end{pmatrix}.$$

The instantaneous eigenvectors ($|\psi_j(t)\rangle$) ($j = 1, 2, 3$) of this Hamiltonian

$$|\psi_j\rangle = \frac{A_j}{\sqrt{M_j}}e^{2i\phi}|\Phi_1\rangle + \frac{B_j}{\sqrt{M_j}}e^{i\phi}|\Phi_2\rangle + \frac{C_j}{\sqrt{M_j}}|\Phi_3\rangle \quad (16)$$

with

$$A_j = -\sin^2(\theta) + 2(-\eta - E_j)(\eta + \cos(\theta) - E_j) \quad (17)$$
$$B_j = -\sqrt{2}\sin(\theta)(\eta + \cos(\theta) - E_j) \quad (18)$$
$$C_j = \sin^2(\theta) \quad (19)$$

and

$$M_j = 2\sin^4(\theta) + 2(\eta + \cos(\theta) - E_j) \times [\sin^2(\theta)(3\eta + E_j + \cos(\theta)) + 2(\eta + E_j)^2(\eta + \cos(\theta) - E_j)] \quad (20)$$

where E_j are the solution of the equation (21)

$$E_j^3 - \eta E_j^2 - (\eta^2 + 1)E_j + \eta + \eta^3 - 2\eta\cos^2(\theta) = 0. \quad (21)$$

Using equation (16) and (13), the Geometric phase of the composite system can be obtained

$$\Gamma_j = 2\pi\frac{-2A_j^2 - B_j^2}{M_j} \quad (22)$$

we can analyze the properties of the geometric phase by using the numerical calculus method. In Figure 5, we plot the geometric phase as a function of η with a fixed θ ($\theta = \pi/3$). From Figure 5, we find that that the geometric phases tend to zero (or 2π) when η increase continuously ($\eta \to \infty$),this is similar to the case that only one subsystem driven by the magnetic field. We also plot in Figure 6 geometric phase Γ_1 as a function of η with different fixed θ, From Figure 6, we find that the geometric phase Γ_1 more slowly reduce to zero when θ increase.

4 Geometric phase in the decoherence system

In this section, using the quantum jump approach, we obtain the correction to geometric phase due to the decoherence when the system evolve in the adiabatic way.

Let us consider the simplest case of decoherence: our system evolving under the Hamiltonian (6) and subjected to spontaneous decay, which can be described by the master equation (1) with $\Gamma = \sqrt{\lambda}\sigma_-^1$, where λ is the spontaneous decay coefficient. Thus, the effective non-Hermitian Hamiltonian in the no-jump case is $\tilde{H} = H - i\frac{\lambda}{2}\sigma_1^z$. Using equations (2) and (3), we obtain the geometric phase for a no-jump evolution

$$\gamma_{1,2} = 2\pi\left\{1 - \mathrm{Re}\left[\frac{\sin^2(\theta)}{\left(C_{1,2} - i\frac{\lambda}{2}\right)^2 + \sin^2(\theta)}\right]\right\},$$
$$\gamma_{3,4} = 2\pi\left\{1 - \mathrm{Re}\left[\frac{\sin^2(\theta)}{\left(C_{3,4} - i\frac{\lambda}{2}\right)^2 + \sin^2(\theta)}\right]\right\}, \quad (23)$$

with

$$C_{1,2} = \cos(\theta) - \eta \mp \sqrt{\eta^2 - 2\eta\cos(\theta) + i\eta\lambda + 1},$$
$$C_{3,4} = \cos(\theta) + \eta \mp \sqrt{\eta^2 + 2\eta\cos(\theta) - i\eta\lambda + 1}. \quad (24)$$

We can also adopt the biorthogonal basis technique [31] to obtain these results corresponding to the real part of the complex geometric phase. Under the adiabatic approximation, in order to consider the influence of decoherence on the geometric phase, we can expand the equation (23) with respect to λ, then retaining the terms up to the second order in λ the geometric phases read

$$\gamma_{1,2}^d \simeq \gamma_{1,2} + 2\pi\frac{\lambda^2}{B_{1,2}^2}\sin^2(\theta)\left\{\frac{[\cos(\theta) + E_{1,2}]^2}{B_{1,2}} - \frac{1}{4}\right\},$$
$$\gamma_{3,4}^d \simeq \gamma_{3,4} + 2\pi\frac{\lambda^2}{B_{3,4}^2}\sin^2(\theta)\left\{\frac{[\cos(\theta) + E_{3,4} - 2\eta]^2}{B_{3,4}} - \frac{1}{4}\right\}, \quad (25)$$

with

$$B_{1,2} = [\cos(\theta) - \eta + E_{1,2}]^2 + \sin^2(\theta),$$
$$B_{3,4} = [\cos(\theta) + \eta + E_{3,4}]^2 + \sin^2(\theta). \quad (26)$$

It is worth pointing out that equation (25) returns to equation (23) when we only consider the terms of the first order in λ. In other word, in the case of low decoherence, equation (25) reveals that the resulting error is of order λ^2/B_k^2 ($k = 1{-}4$) and thus can be made negligible by making B_k large enough. The results clearly show that the lowest correction due to the decoherence for the "no-jump" trajectory is only of second order in the decaying coefficient λ. This reflects the resilience of the geometric phase against the environment, in other words, the decoherence resource can cause this system to jump from the one instantaneous eigenstates into the other instantaneous eigenstates, but this process is restricted by the adiabatic approximation. For more general condition, the state of a system may stay at the instantaneous eigenstates of the instantaneous Hamiltonian under the adiabatic limit, we can say that this property opposes the tendency of the environment of dragging the state away from its undisturbed evolution.

Sjöqvist [26] has defined a meaningful geometric phase for an open system and proposed an experiment scheme to implement the geometric phase for an individual trajectory. This idea may be useful in understanding our results or designing a corresponding experiment. we noted that Yi [29] has deal with the quantum jump approach to an open bipartite system, which could be helpful in understanding our paper.

5 Conclusions

In conclusion, We use the numerical calculus method to investigate the geometric phase of a composite system which is composed of two spin-$\frac{1}{2}$ particles driven by a classical magnetic field. The results clearly show that the geometric phase dependence on the exchange-interaction coefficient η and polar angle θ of magnetic field and the geometric phase tend to zero (or 2π) when η continuously increase ($\eta \to \infty$) or θ slowly increase from 0 to π. We also consider the special case that only one subsystem driven by time-varying magnetic field. Using quantum jump approach, the influence of simple decoherence on the geometric phase was considered. It is interesting that the lowest correction of the geometric phase due to decoherence is only quadratic in the decoherence coefficient λ. This results reinforce the idea that geometric phases can be robust to errors, in agreement with the previous works which analyze the geometric phase under quantum or classical noise [22,23,30]. Obviously, the applications of geometric phases have been studied in lots of references [16,31], and the corresponding experiment has been proposed by Carollo [32]. We are therefore hopeful that the theoretical ideas presented here may stimulate corresponding experimental activity.

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